Netw Spat Econ (2010) 10:125–145
DOI 10.1007/s11067-007-9035-6

Determining Optimal Police Patrol Areas with Maximal Covering and Backup Covering Location Models

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Published online: 17 October 2007

Abstract This paper presents a new method for determining efficient spatial distributions of police patrol areas. This method employs a traditional maximal covering formulation and an innovative backup covering formulation to provide alternative optimal solutions to police decision makers, and to address the lack of objective quantitative methods for police area design in the literature or in practice. This research demonstrates that operations research methods can be used in police decision making, presents a new backup coverage model that is appropriate for patrol area design, and encourages the integration of geographic information systems and optimal solution procedures. The models and methods are tested with the police geography of Dallas, TX. The optimal solutions are compared with the existing police geography, showing substantial improvement in number of incidents covered as well as total distance traveled.

Keywords Maximal covering · Backup covering · Optimization · Geographic information systems · Police patrol areas · Police beats

Virtually all police departments create administrative and patrol geographic divisions, and their provision of services is influenced by this police geography. Optimal spatial divisions can efficiently distribute limited police manpower and

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other resources, reduce response times, and save money through efficient deployments.

This research investigates how solutions generated with maximal covering models can increase the level of police service by finding more spatially efficient allocations of law enforcement resources under varying scenarios. In doing so this paper makes two additional contributions: we develop a method for integrating geographic information systems (GIS) with linear programming optimization to generate and display alternative optimal solutions, and we formulate an innovative backup coverage model that is appropriate for police patrol area design. Given that the available police resources can change over time, and that emergency situations can alter the demand for police service, these methods and models are required to allow police administrators to determine the optimal police geography under alternative scenarios. These contributions allow police administrators to combine their existing GIS software and expertise and their spatially informed incident database with integer programming solution software in order to flexibly design police patrol and administrative districts.

The following section reviews two pertinent areas of literature: the design of police areas, and the use of maximal covering models in the provision of services. A formulation of the Police Patrol Area Covering (PPAC) model is then provided. This formulation is used to optimally solve several problem instances using crime data and police geographic boundaries for Dallas, TX. Numeric results are presented and efficiency measures are used to compare these results with the existing police geography. Since backup, or multiple, coverage of incidents is of interest to police administrators a variant formulation and method is provided that will solve for maximal backup coverage. A range of near optimal solutions that tradeoff maximal coverage and maximal backup coverage is presented.

1 Literature review

1.1 Location science and GIS in law enforcement

The division of an area by a police force is fundamentally a geographic problem. Commonly, a city is divided into police command areas (e.g., precincts, districts, divisions, etc.) and patrol areas (e.g., beats, sectors, reporting areas, etc.) (Larson 1978; Moonen 2005). Figure 1 shows three levels of the police geography for the city of Dallas, TX. Historically, the police geographic boundaries are hand-drawn based on an officer's or administrator's knowledge of the total area to be patrolled by the police force and the available police resources (Mitchell 1972; Taylor and Huxley 1989). In some cases the boundaries have been drawn such that they respect natural boundaries, they focus on hotspots of crime, or they conform in some way to other administrative boundaries (such as census geography) (Curtin and Hayslett-McCall 2006). In most cases there is no quantitative method for evaluating how the hand drawn boundaries compare to an optimal arrangement. This pervasive and persistent lack of formal procedures for police patrol area development can complicate higher-level policy decision-making due to the lack of objective quantitative measures of efficiency (Taylor and Huxley 1989).

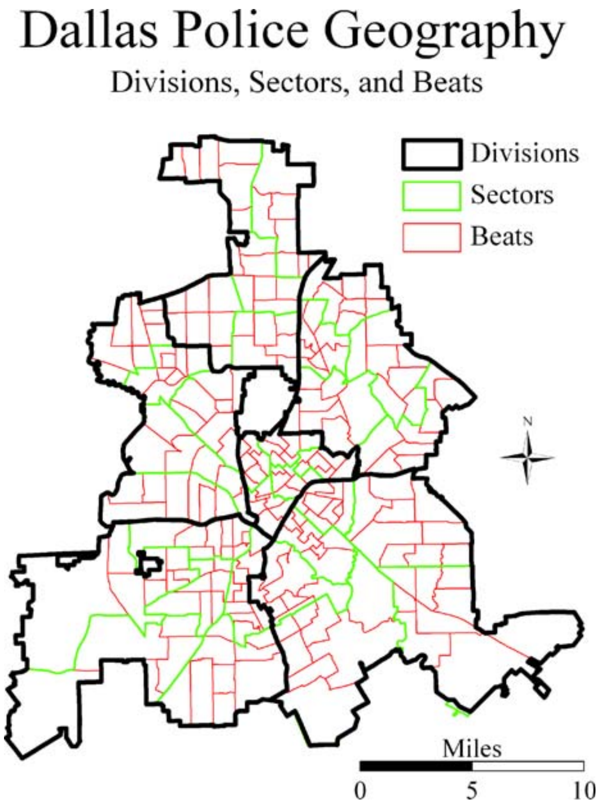

Fig. 1 City of Dallas police geography

Where operations research (OR) techniques have been used in law enforcement, the results of these analyses are sometimes the only quantitative information provided to decision makers (Aly et al. 1982). In the context of police geography the applications of OR techniques can be loosely grouped into two areas: the design of patrol areas, and the deployment of officers within those areas. There has been a paucity of research in the first of these areas, and the present paper seeks to address this deficiency. Notable exceptions exist. The first is an application for Anaheim, CA where a formulation of the *p*-Median problem was employed to minimize total weighted travel distance to service the expected calls (Mitchell 1972). Although no proven optimal solutions were found, heuristic solutions based on the Maranzana heuristic (Maranzana 1964) resulted in a 13 to 24% reduction in average response distance when compared to hand drawn districts. Another research effort also used a distance minimization formulation with an interchange heuristic to allocate police briefing stations to districts by shift (Aly and Litwhiler 1979). A third example of OR techniques in police administration used a simulation method to design patrol areas such that the amount of time available for repressive patrol was maximized (Carroll and Laurin 1981).

In contrast to patrol area design, there have been several successful research efforts regarding the allocation or deployment of officers among patrol areas. The Rand Corporation supported a series of research publications on a hypercube queuing model for police deployment (Larson 1975). This model was designed to find deployment patterns *for a pre-determined set of police patrol areas*. A more advanced Patrol Car Allocation Model was developed (Chaiken and Dormont 1978a, b) and distributed to police departments (Chaiken 1978). This model was updated to include multiple dispatch queuing (Green 1984; Green and Kolesar 1984a), and its validity was tested with generally positive results given the limitations of a model dependent on human behavior (Green and Kolesar 1989). Although much of this work has been focused on New York City, variations of queuing models have been applied widely, including in St. Louis County, MO (Kwak and Leavitt 1984) and New Britain, CT (Sacks 2000). Other research has examined strategies for wide area (rural) patrol (Birge and Pollock 1989), and allowed user specified sector design and deployment strategies to be tested (Kern 1989). Moonen (2005) provides a review that demonstrates how deployment models have repeatedly been used to assess the quality of district design, without suggesting methods for optimally determining those designs.

Given the combinatorial complexity of the police districting problem, it is unlikely that an optimal districting solution will be chosen by chance, that can subsequently be submitted to a deployment solution procedure. There has been relatively little work regarding optimal emergency planning strategies in the past 15 years, with even less directed at the optimal determination of patrol areas, and calls have been made for the development and implementation of such models (Green and Kolesar 2004). In this paper, optimization models and solution procedures are developed that allow police patrol areas to be designed based on the objectives of maximal coverage and maximal backup coverage.

In contrast to OR techniques, GIS have become widely accepted among police departments as a valuable tool (Harries 1999). One significant example of this is the use of GIS in the determination of clusters of crime activity (i.e. hot-spots) (Craglia et al. 2000; Harries 1999). However, GIS are generally not capable of solving combinatorially complex location science problems optimally (Church 2002). In fact, only a very limited number of problems can be solved and these can only be solved heuristically, generally using versions of interchange heuristics (Teitz and Bart 1968; Zanakis et al. 1989). Unfortunately, these heuristics can operate in "a minefield of local optima" (Church and Sorenson 1994), which can lead to substantially suboptimal solutions. When heuristics are employed in a GIS, there is no way of knowing whether or not the optimal solution has been determined, or how close the solution is to optimal. Although there are superior heuristic procedures (D'Amico et al. 2002) these heuristics require sophisticated users to test and apply parameters. An earlier research project demonstrated that GIS and linear programming solution software could be effectively integrated in order to allow police administrators to solve optimization models (Curtin et al. 2005). The present research goes well beyond that proof-of-concept effort and employs an innovative combination of GIS analysis techniques and maximal covering formulations to determine optimal police patrol areas.

1.2 Covering models

The Maximal Covering Location Problem (MCLP) was first formulated in the mid-1970s (Church and ReVelle 1974). The MCLP seeks to find the solution to the problem of locating facilities that maximizes the coverage of demand for services within a given acceptable service distance (or response time). Because the MCLP has been shown to be extremely combinatorially complex, a series of heuristic solution procedures have been developed (AdensoDiaz and Rodriguez 1997; Galvao et al. 2000; Galvao and ReVelle 1996). Additionally, the MCLP can be seen as a variant formulation of other prominent location models including the *p*-Median model and the Location Set Covering model (Church and ReVelle 1976), and research has been conducted on the effects of data aggregation errors on solutions (Current and Schilling 1990). Variations of the MCLP have been formulated to include workload capacities (Pirkul and Schilling 1991), or to maximize coverage and minimize distances to demands outside the maximum covering distance (Church et al. 1991). Several models that include conditional coverage (ReVelle et al. 1996), backup coverage (Hogan and ReVelle 1986), or both (Pirkul and Schilling 1988) have appeared, and a more thorough treatment of this literature appears in Section 3.2 below.

Covering models have been applied to the location of emergency warning sirens (Current and Okelly 1992), the location of ambulance bases in rural areas (AdensoDiaz and Rodriguez 1997), integrated fire and ambulance siting (ReVelle and Snyder 1995), the location of retail facilities (Berman and Krass 2002), and ecological reserve selection (Church et al. 1996). A review of applications of the MCLP that do not involve geographic location (Chung 1986) found that the model was proven useful for data abstraction and statistical classification. To date, no application of the MCLP to the determination of police patrol or administrative areas has appeared in the literature. In the context of this research, it is shown below that the implementation of covering models can result in savings in terms of cost of operations (i.e., fuel costs) and decreased response times based on the more efficient spatial arrangements.

2 The Police Patrol Area Covering (PPAC) model

Maximal covering models can be applied to the problem of generating optimal police patrol areas with the following formulation:

$$\text{Maximize } Z = \sum_{i \in I} a_i y_i \tag{1}$$

Subject To:

$$\sum_{j \in N_i} x_j \geq y_i \text{ for all } i \in I \tag{2}$$

$$\sum_{j \in J} x_j = P \tag{3}$$

$$x_j = (0, 1) \text{ for all } j \in J \tag{4}$$

$$y_i = (0, 1) \text{ for all } i \in I \tag{5}$$

Where:

I, i = the set and index of known incident locations or calls for service
J, j = the set and index of potential locations for police patrol command centers
S = the acceptable service distance (surrogate for desired response time)
d_{ij} = the shortest distance from incident location i to police command center location j
x_j = 1 if a police patrol is located at potential site j, and 0 otherwise
y_i = 1 if an incident location at i is covered by at least one located police patrol area, and 0 otherwise
N_i = $\{j \text{ in } J \mid d_{ij} \le S\}$
a_i = weight or priority of crime incidents at incident location i
P = the number of police patrol areas to be located

In this formulation N_i is the set of command or dispatch centers eligible to provide "cover" to incident location i. In the context of patrol area development, N_i is the set of potential command centers or patrol area centers that could respond to incident i within the acceptable response time, S. S can vary for different types of incidents or different response time priorities. Keep in mind that although d_{ij} and S do not appear directly in the formulation, they are included in constraints (2) through the inclusion of the sets N_i. The objective is to maximize the number of weighted incidents served or "covered" within the acceptable response time. Any subset of crime incidents may be used to populate the set I. For example, if there are seasonal trends in crime incidents, it may be appropriate when defining patrol areas for a given week (or month) to consider just those incidents that occurred during the same week (or month) of the previous year.

Constraints of type (2) allow y_i to equal 1 only when one or more patrol cars are established at sites in the set N_i. The number of patrol areas to designate (P) is user-defined, and could, for example, be limited to the number of available patrol cars. This limit is enforced by constraint (3). Constraints (4) and (5) require that only integer values are included in the solution. That is, police patrols cannot be split between patrol areas.

The PPAC model assumes that an acceptable level of service (measured as a response *distance*) has been agreed upon as representing an acceptable level of citizen safety. This assumption is reasonable, as police response time can be a significant determinant in the evaluation of police performance (Bodily 1978; Green and Kolesar 1984b), and response time is highly correlated with response distance (Priest and Carter 1999). However, the integration of GIS with the optimization solution software allows any appropriate impedance value (or values) to be used in the computation of the "distances" between incidents to be covered and the facilities which may cover them. The GIS maintained by the police administration may contain information regarding street width, speed limit, travel times, congestion conditions, turn impedances, or other factors influencing response times. Moreover, these factors may change temporally, and administrators may wish to evaluate the coverage of incidents under a variety of road conditions. Although research has shown that response times have little bearing on the volume of crime in a jurisdiction (Sherman et al. 2004), police departments are subject to a number of resource constraints and political realities that make the efficient delivery of services important. Given the limit on police resources, the implementation of PPAC also requires that the number of police patrols is known in advance. This is, in fact, one of the models' strengths given that the amount of financial resources to be allocated to police protection may change quickly and often.

As is customary with many location models, assumptions are made within PPAC regarding the location of the police patrols and the locations of central facilities. This command center is the location where administrative decisions over crime patterns in the coverage area are made; and this site is often termed the division or sector headquarters. In terms of patrol areas, this central location is the place from which police patrol cars are most likely to be dispatched. While it is certainly the case that sometimes patrol officers respond to calls while they are on active patrol within their designated patrol area, there is no way to know in advance their exact location in order to determine the precise coverage properties of that patrol car. Under these circumstances the central location becomes the best assumption of their position within the patrol area. Although one could select random locations within areas to simulate the probabilistic nature of police car locations, any spatial measure of central tendency of these randomly chosen locations would converge on a central location such as the centroid of a polygon. Note also that the assumption of dispatch from a central location is perfectly acceptable to both police administrators and patrol officers who have been consulted, while the modeling of random locations for police patrols are unanimously regarded as unrealistic.

3 Solving instances of the PPAC model

3.1 Classic covering for police applications

The maximal covering model defined above was solved using crime data from the Dallas Police Department (2002b) along with the hierarchical geographic administrative and patrol boundaries (Fig. 1) for the city of Dallas (Dallas Police Department 2002a). Alternative optimal arrangements of the police geography at three levels were generated and the improvements over the existing arrangements were documented. Presented here are the results obtained when determining the optimal arrangement of sectors within each of the divisions of the Dallas Police geography. The boundary of the North Central Division with the current sector and beat boundaries contained within it are shown in Fig. 2, along with the 267 geocoded calls for police service within that division for a single day (07/20/2000). Although there is a concern for the accuracy of address information captured by police officers, a geocoded address is likely to be the best available spatial representation for these incidents.

The data shown in Fig. 2 were used as the basis for determining the arrangement of five sectors within the North Central Division that would maximally cover the

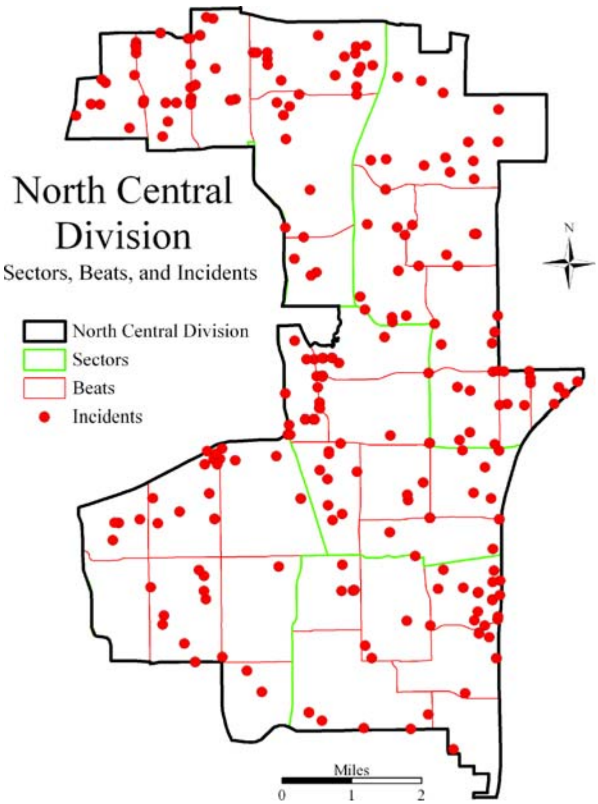

Fig. 2 North Central Division Sectors, beats, and incidents

weighted incidents within it. The weights on the incidents were based on the Signal Code variable used by the police department. These values communicate the priorities assigned to different calls to the police officers on patrol, and define procedures for their response. The calls range from extremely serious crime incident locations (murders, armed robberies, officer down, etc.), to much less serious incidents such as vandalism reports or minor (noninjury) car accidents.

The incidents themselves represent the set of locations i that should be covered, and the Signal Code weights correspond to the a_i values in the PPAC formulation given above. This process was repeated for each of the six police divisions. The service distance S was chosen to be two miles for each division, with the exception that S was defined as one mile for the markedly smaller Central Division. The 2-mile service distance was chosen based on the researchers' observation that most beats in the study area had cross-beat distances of between one and two miles. The presumption is that a patrol car in a beat should be within the covering distance of any incident within that beat. Unfortunately there is no single, well-accepted value for an acceptable police response distance or time (Hill 2006). There are, in fact, many subjective and objective measures of police service provision (Brown and Coulter 1983), and response time is only one of many. Acceptable response times are context dependent, where the context includes the nature of the incident, the available police resources, and equity of service provision. Perhaps most importantly, variation in local geographic patterns (high-rise urban vs dispersed rural populations) create an extraordinarily wide range of acceptable service distances (Kane 2006). Moreover, the relationship between road network distance (particularly as measured in a GIS) and emergency response time is not well understood (Marble 2006), although federally sponsored research in this area is underway (Levine 2006). An advantage of the research presented here is that any S can be chosen for the solution of PPAC, and in fact multiple values of S can be tested in order to determine the sensitivity of the optimal solution to changes in that parameter. The lack of a well-accepted standard demands this flexibility from any research effort in this area.

The origin–destination (OD) matrix was generated based on an all-to-all shortest-path algorithm, and the sets N_i were generated through the use of a custom selection and query interface. This process is demonstrated in Fig. 3 where the shortest distances from a single incident to all potential command center sites are displayed, and the paths to the two facilities that are within distance S are highlighted. This set-generation process was performed for each of the 267 incident locations in order to generate the sets N_i. The underlying road network used in this process is the highest quality available for the area, developed by the North Central Texas Council of Government. The centroids of beats within the division comprised the potential facility sites.

Based on the results of the process described above, the information necessary for solving the PPAC problem were exported to the linear programming solution software. This information included the number of potential facility sites, the number of incident locations, the weight values for each incident (a_i) and the sets N_i. Optimal solutions were generated using the ILOG Optimization Programming Language (OPL) Studio and CPLEX 8.1 optimization software for integer programming applications. This software combines the use of a version of the simplex solution method (Dantzig 1957) on linear programming relaxations of the problem, with a complementary branch and bound technique for dividing the original problem into more solvable subproblems (Hillier and Lieberman 1995).

In our example, the optimal solution consisted of the five locations (beat centroids) that would best serve to cover the weighted incidents in the North Central Division. Those locations and the routes to the incidents that they would likely serve are shown in Fig. 4. It is a straightforward procedure to assign beats to new sectors based on the incidents served by each optimal facility location, in such a way that the hierarchical police geography is preserved. It should be noted, though, that the need to preserve the geographic hierarchy results in some beats being assigned to a sector, while some incidents within that beat may be closer to a neighboring sector command center. This is shown in Fig. 4 where routes to incidents cross sector boundaries. If the beat boundaries could be redrawn this issue would become moot. With the generation of the optimal solution it is possible to compare its level of service with the existing sector arrangement. Table 1 shows the results of this comparison for each of the six police divisions.

In terms of total distance traveled between the command center locations and the incidents to be served, the existing arrangements required 4,398.9 total miles of

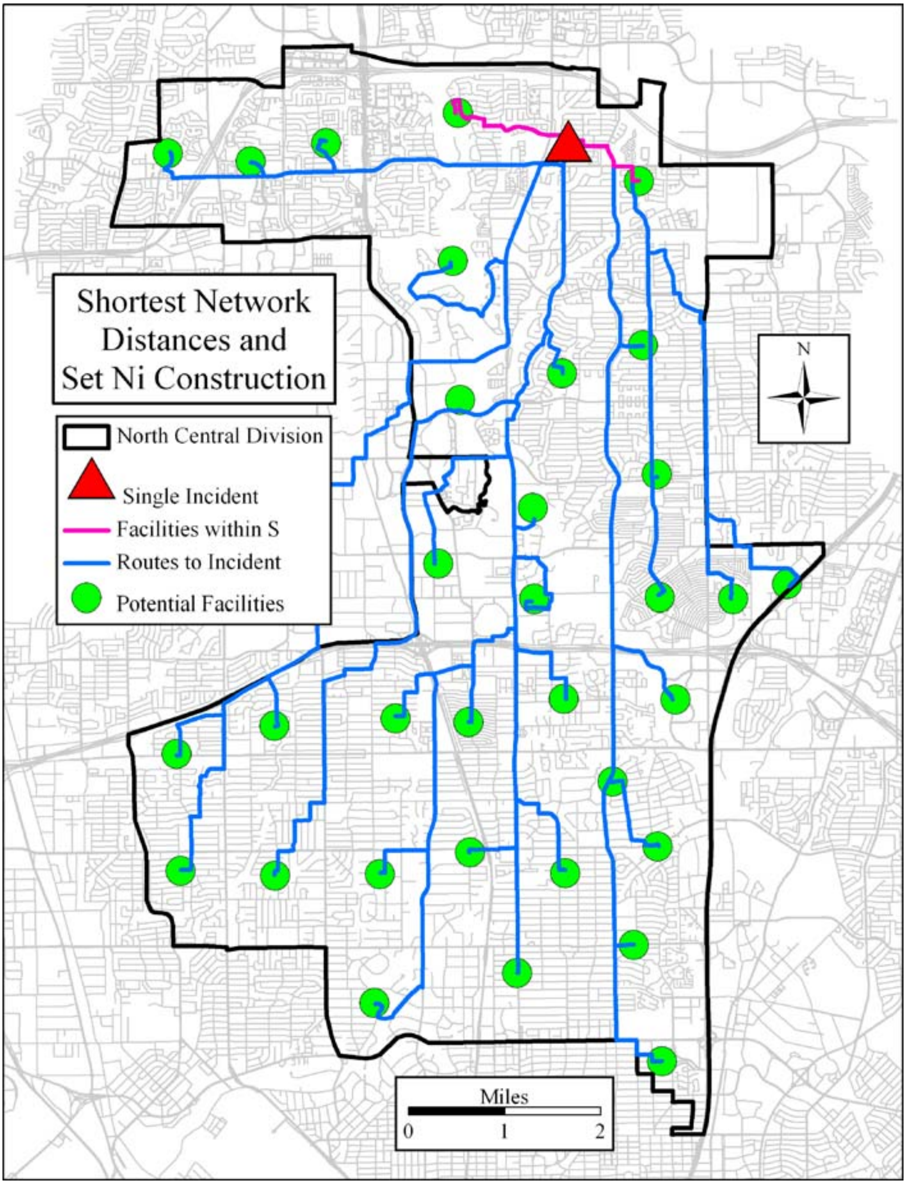

Fig. 3 Shortest network distances and set N_i construction

police travel, while the new solution required only 3,657.9 miles of police travel; an improvement of 18.9%. Perhaps more importantly, for the total patrol area the optimal solutions could cover substantially more incidents within the selected service distance *S* (78.9%−60.9%=18% improvement).

The results show remarkable improvements in the ability of police to respond to calls for service. The nearly 19% reduction in total distance traveled by police officers could dramatically improve response times and reduce costs (particularly

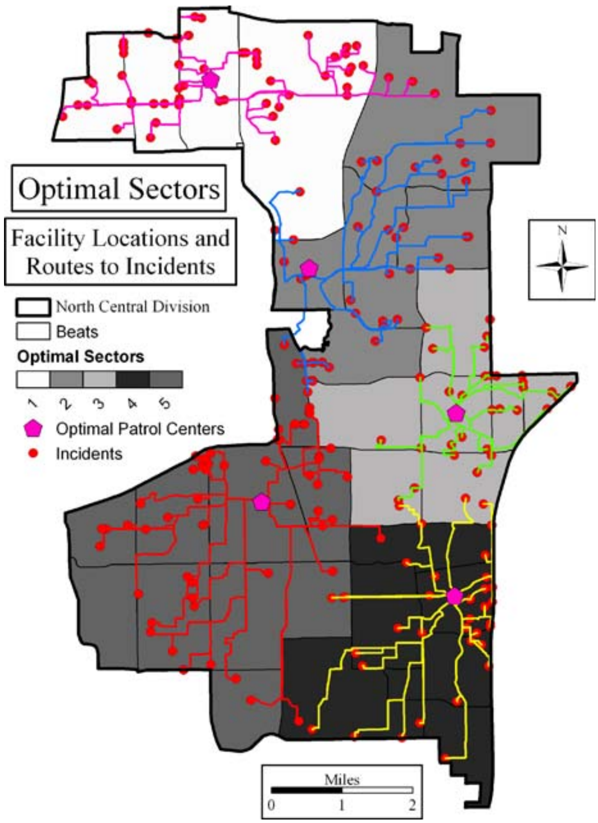

Fig. 4 Optimal sectors and routes to incidents

fuel costs). Police administrators take notice of the magnitude of these types of improvements in efficiency.

The calls-for-service from a single day are used in the practical demonstration above, and such an analysis would be appropriate under conditions where a significant event occurred on that day, such as a natural or man-made disaster, or a single-day staffing emergency (a walkout or strike). However, police administrators may well want to plan patrol boundaries for an extended event (such as a festival or

Table 1 Police efficiency measures with the existing and optimal spatial arrangements

Division	Existing total miles	Optimal total miles	Decrease in total distance (%)	Existing percent of calls covered within S (%)	Optimal percent of calls covered within S (%)
Northwest	625.0	534.2	14.5	71.1	83.2
North central	616.7	446.7	27.6	45.7	73.0
Northeast	811.0	760.4	6.2	66.7	78.3
Central	252.3	230.0	8.8	72.6	83.4
Southwest	841.3	727.2	13.6	59.1	78.7
Southeast	1,252.6	959.4	23.4	50.3	76.8
Total area	4,398.9	3,657.9	18.9	60.9	78.9

large convention) using the calls-for-service from an entire week. They may wish to change patrol boundaries on a monthly basis, or they may want to change boundaries with the seasons, since it is known that frequency and type of crime varies with the seasons. In order to test such scenarios, results were obtained for a range of time periods up to one year (Table 2), with as many as 87,603 calls-for-service being employed in the analysis. The largest of these problem instances was solved in less than one hour on a desktop workstation running under Windows XP Professional with a 2.13 GHz Pentium processor and 2.0 GB of RAM. In addition to altering the time period for analysis, police administrators may want to vary the set of calls-for-service to include only those types (or priorities) of calls which are determined to be most important in constructing particular district designs.

3.2 Encouraging backup coverage

Although the maximal covering solutions certainly appear promising regarding the increases in efficiency they could encourage, these solutions are only optimal if one accepts that maximal coverage is the only objective to consider, and if both the constraints and the data are representative of the real conditions under which the police must operate. Since we know that the model and the datasets are necessary simplifications of reality, it may well be profitable to explore additional solutions that allow administrators or other decision makers more flexibility in police patrol area design. In order to provide these alternatives the notion of backup-or multiple-coverage is presented.

The concept of multiple coverage in the context of emergency service provision was introduced as a secondary objective for the Location Set Covering Problem (LCSP), which seeks to minimize the number of facilities that will cover all calls for service within an acceptable service distance (Daskin and Stern 1981). In this context, the secondary (multiple coverage) objective is designed to select from among the alternative optima for the classical LCSP. This model provides not a tradeoff of objectives, but rather a hierarchical set of objectives, where multiple coverage is decidedly secondary. Their notion of backup coverage has been extended from this model to include dual-objective LSCP and MCLP models that encourage double (not multiple) coverage (Hogan and ReVelle 1986). Hogan and Revelle (1986) also suggest that additional coverage (3rd, 4th, etc.) could be incorporated by

Table 2 Problem instances solved from 1 week to 1 year time periods

Time period	Number of calls
1 year	87,603
Winter	20,212
Spring	22,692
Summer	22,494
Fall	22,205
January	6,790
July	7,220
Week (August)	1,796
Week (December)	1,881

continuously adding additional objectives, and employing weighting schemes to generate tradeoffs among them. Another model has appeared in the literature that seeks to minimize the cost of providing primary and secondary (backup) coverage with additional workload constraints (Pirkul and Schilling 1988).

In the context of police patrol area design, backup coverage is achieved when more than one patrol car can cover an incident within the service distance. Depending on the priority of the incident and the nature of the call for service, the police may want multiple patrol cars to be available within the service distance. Therefore, it may be that any number of located facilities (up to P) should cover a single incident. Although this is more closely related to the notion of "multiple coverage" as it appears in the literature, we refer to it as backup coverage due to the accepted definition of that term among police administrators.

Although multiple coverage could be achieved through formulations that employ multiple objectives, the PPAC model can be made to encourage backup coverage very simply by replacing constraints (5) in the formulation above with:

$$y_i \in \{0, 1, \ldots, P-1, P\} \text{ for all } i \in I \tag{6}$$

allowing the variables y_i to take on integer values from 0 to P rather than restricting them to 0 or 1. Since the objective function increases every time a variable y_i increases, this encourages maximal backup coverage of incidents. Moreover, since a_i values increase as call priority increases, this variant formulation encourages backup coverage of the most important calls. This backup formulation differs from what has appeared in the literature in several significant respects. Most importantly, backup coverage is not a secondary goal; coverage of any kind is the goal and backup coverage is more valuable by its nature than single coverage. Moreover, there are no diminishing marginal returns for each successive facility that covers a demand. The nth covering is worth as much to the objective function as the first. Since backup coverage is just a more valuable form of coverage, there is no need for multiple objectives in the formulation, or for the addition of multiple types of coverage variables, or for the addition of constraints to enforce values of those variables. The complexity of the formulation is not increased by simply relaxing the range of acceptable integer values for y_i. This type of relaxation of variable values has been employed in the context of the LSCP where multiple coverage was a secondary objective (Daskin 1995).

Unfortunately, a solution that encourages maximal backup coverage without enforcing maximal coverage will tend to give solutions where the facilities (police patrol centers) are simply clustered around the most serious incidents. Figure 5 contrasts the maximal covering solution with the maximal backup covering solution for the Southeast Division of the Dallas Police geography. As can be seen in Table 3, the maximal backup coverage problem instance (Solution # 1) covers only 170 incidents within the two mile service distance, although all but nine of those incidents are covered by more than one facility, and 66 incidents are covered by all six of the located facilities. Clearly the maximal backup solution concentrates resources too heavily in one—albeit high-crime—area, at the expense of the rest of the division. Although such a solution is not appropriate for police patrol area delineation, it may be of use in other police deployment contexts, including tactical response. Additional research is being undertaken to explore models that are appropriate for a variety of deployment needs.

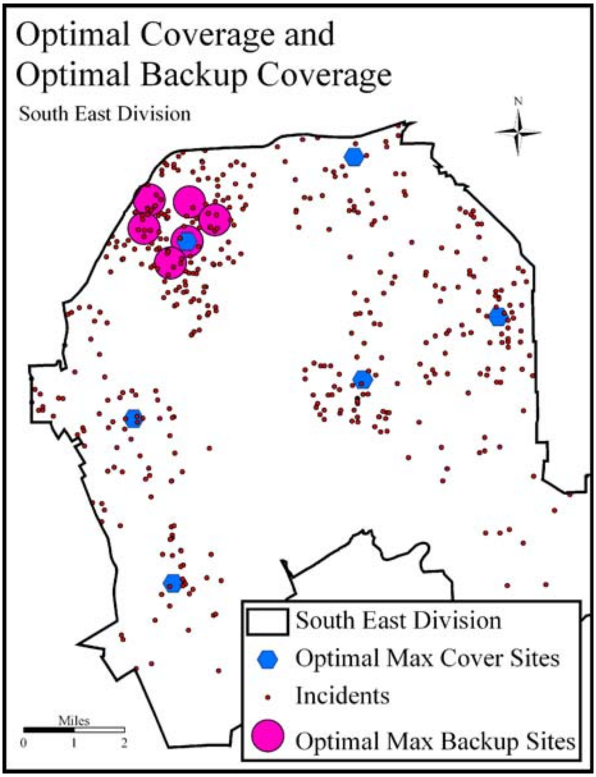

Fig. 5 Maximal coverage vs. maximal backup coverage

Conversely, the maximal covering solution (solution # 24) covers 416 incidents within the service distance, but none of these incidents are covered by more than a single facility. This suggests that a tradeoff between these two objectives may give alternative optimal solutions that are superior to either of the extremes.

3.3 Multiobjective results

Such tradeoffs can be generated by solving first for the maximal covering objective to obtain the upper bound on maximal coverage. Then the bounds of y_i are relaxed as discussed above to encourage backup coverage. A set of decision variables (w_i) is added that is defined just as y_i had been in the maximal covering formulation. A single constraint is then added to mimic the original maximal covering objective function:

$$\sum_{i=1}^{n} a_i w_i \geq O \tag{7}$$

where O is the value less than or equal to an upper bound limit on the maximal covering objective. That is, this constraint ensures that a minimum level, O, of covering will be enforced by the user. Finally, this model is solved repeatedly for a range of values of O to determine the tradeoffs between maximal backup coverage

Table 3 Maximal backup coverage vs maximal coverage

Solution number	Maximal backup objective	Maximal covering objective	Total incidents covered	Covered once	Covered twice	Covered 3 times	Covered 4 times	Covered 5 times	Covered 6 times
1	2,279	495	170	9	10	18	30	37	66
2	2,268	500	172	10	11	21	26	43	61
3	2,210	743	244	83	16	21	50	74	0
4	2,154	748	246	85	17	28	55	61	0
5	2,085	774	253	92	18	39	61	43	0
6	2,037	930	296	140	18	53	85	0	0
7	2,007	932	297	143	24	43	87	0	0
8	1,951	937	299	147	26	55	71	0	0
9	1,902	941	298	144	21	90	43	0	0
10	1,882	959	305	152	39	62	52	0	0
11	1,848	1,019	326	160	52	114	0	0	0
12	1,847	1,082	347	199	34	114	0	0	0
13	1,827	1,112	359	212	48	99	0	0	0
14	1,757	1,114	360	223	50	87	0	0	0
15	1,701	1,119	362	231	60	71	0	0	0
16	1,692	1,123	361	216	93	52	0	0	0
17	1,642	1,137	366	221	116	29	0	0	0
18	1,632	1,141	368	248	68	52	0	0	0
19	1,601	1,215	388	255	133	0	0	0	0
20	1,592	1,219	390	261	129	0	0	0	0
21	1,541	1,245	400	297	103	0	0	0	0
22	1,532	1,249	402	303	99	0	0	0	0
23	1,307	1,296	415	412	3	0	0	0	0
24	1,298	1,298	416	416	0	0	0	0	0

and pure maximal coverage. This solution process is one implementation of a technique generally known as the constraint method of multiobjective programming.

The results of completing this process for the Southeast Division are shown in Table 3 and Fig. 6. Table 3 shows that there were 24 unique optimal solutions generated with increasing enforcement of the maximal covering objective. Figure 6 shows the Pareto optimal tradeoff curve between these two objectives.

With increasing enforcement of the maximal coverage objective, *O*, backup coverage decreases until the objective function values converge (at a value of 1,298). This graphic could be of significant benefit to police administrators since it highlights solutions along the continuum from maximal coverage to maximal backup coverage that perform better relative to other solutions close to them in objective space. As an example, solution number 22 (in Table 3) may be considered much more desirable than solution 24, since coverage of only 14 incidents is lost, while 99 incidents gain double coverage. In fact, these two solutions are similar in that they share four of the same facility locations (Fig. 7).

4 Refinements

There are several avenues for continued improvement of PPAC. These can be loosely grouped into data-based issues and formulation issues. Regarding data

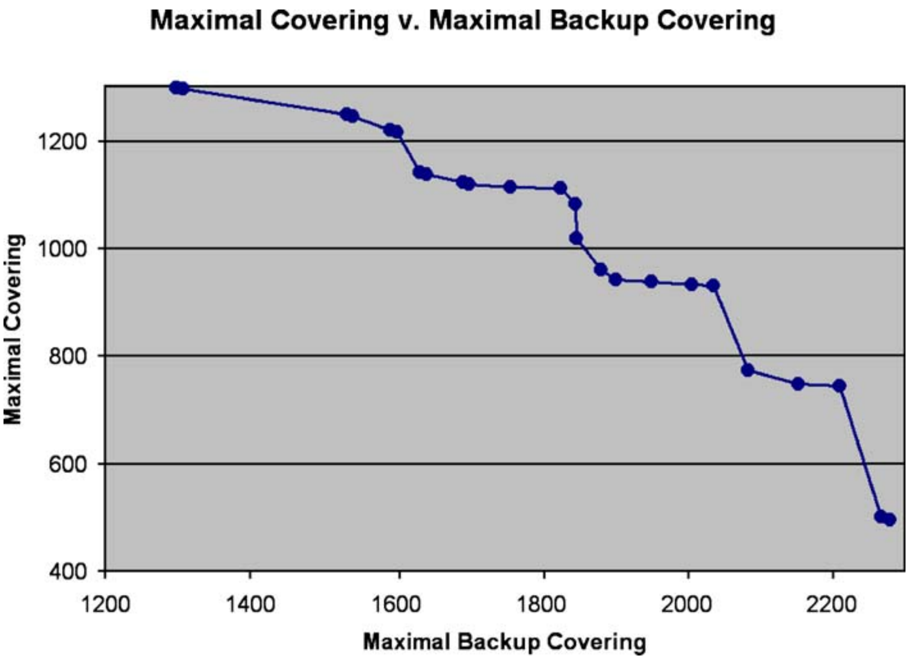

Fig. 6 Objective function values for maximal coverage and maximal backup coverage

issues, more work is needed to determine the nature of the weights based on the Signal Code priority scheme. The values of Signal Codes range from 1 to 5 (with 5 having the highest priority) but there is no documentation on the relative weight of those values. It cannot necessarily be stated that a call for service with a priority value of 5 is five times more important than a call for service with priority 1. That, in fact, is almost certainly not the case since calls with a priority of 5 are frequently life-threatening, while calls with a priority of 1 are often of negligible concern for the police. Therefore, the priority function is not linear, and additional information regarding relative priority values must be supplied by police experts. A second data issue involves the set of potential command center locations. There is no currently available set of such locations, so the centroids of the next lowest level in the police hierarchy have been used as the best substitute set. It may be that cadastral data showing vacant parcels or buildings under consideration as command centers could be used instead. Third, it is at this time unknown what limit there will be on the magnitude of the sets of calls for service and potential facility locations that will still allow for optimal solution. Even using the expanded address-geocoded data presented above, all model instances that have been solved up to now have not presented a challenge in terms of required solution time or other resources. If a limit of feasible solution is found, this will influence the type and frequency of questions that can be answered regarding optimal police area design.

The second group of refinements involves variations of the model formulation. It is presumed that there are a large number of other potential constraints—physical resource constraints, economic constraints, legal constraints, scheduling constraints—that could be included in the formulation. The resource constraints may limit the

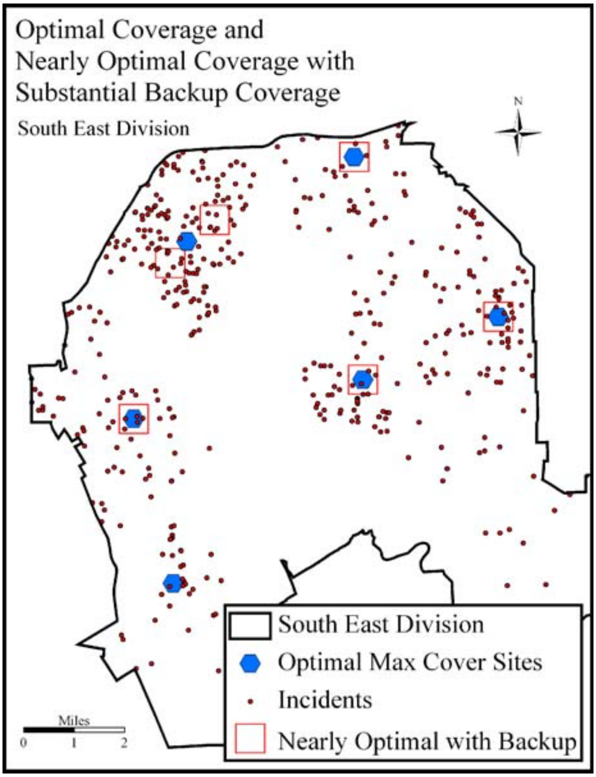

Fig. 7 Alternative solutions – maximal coverage vs. nearly maximal coverage with substantial backup coverage

number of patrol cars, police officers, or other available staff. Economic constraints related to the city budgets are almost certainly a concern. It is presumed that there are numerous legal constraints on the level of police service that must be provided by the police, and that control the areas that they must patrol. Based on a survey of crime mapping professionals, it is noted that scheduling constraints are frequently imposed by the contracts of the police officers and their union, and thus there is a strong motivation to distribute the workload among the officers so that no patrol is assigned to a disproportionate number of calls, weighted by priority as a proxy for severity and risk. Constraints that enforce a capacity on the weighted number of calls could take the following form:

$$\sum_{i \in N_j} a_i x_j \leq M_j \text{ for all } j \in J \tag{8}$$

where:

N_j $\{i \text{ in } I \mid d_{ij} = S\}$
M_j the maximum incident load that a patrol area centered at j can serve;

These constraints differ from those formulated by Pirkul and Schilling (1991) in two respects. First, constraints (8) employ a service neighborhood rather than a

Table 4 Worst case distances—optimal vs existing

Division	Optimal solution worst case distance (ft)	Existing arrangement worst case distance (ft)
Southeast	43,235	30,675
Southwest	37,611	23,121
Northwest	20,051	19,933
Northeast	20,092	22,530
North central	23,239	27,628
Central	9,528	7,622

binary coverage variable. The set N_j is defined as all of the crime incident sites (i) that can be served from a potential patrol area centroid (j). There is one constraint for each potential patrol area centroid (j). Second, the capacity constraints are based on the siting variable (x_j) rather than the coverage variables. If a patrol area is centered at j the value of x_j will be 1. If this is the case the constraints will require that the sum of the crime incident values (a_i) for all of the sites i that are covered by j must be less than or equal to the maximum crime incident load that can be handled by that patrol area.

The fact that the worst-case distance (the furthest distance from any incident to its nearest facility) generated by the optimal PPAC solutions was longer for four of the six Dallas divisions (Table 4), suggests another formulation refinement. A bicriterion maximal covering formulation that attempts to maximize coverage while simultaneously minimizing the total weighted distance to uncovered incidents (Church et al. 1991) may ameliorate this situation.

Lastly, while patrol and administration are the most commonly cited uses of police districts according the crime mapping professionals, there are other location models that may more appropriately be used for targeted police activities. For example, p-median problems may be best for targeting hotspots of crime, while flow-covering models may be the best for the location of speed traps or drunk driving checkpoints. Research along these lines is continuing.

5 Conclusions

Although police geographic boundaries are defined by virtually every police department, there is a paucity of quantitative methods for evaluating these spatial divisions. This paper demonstrates that police patrol and administrative areas can be optimally delineated through the use of maximal covering models. The integration of GIS and linear programming provides a practical method through which alternative spatial arrangements can be presented to decision makers. A test of this method using the police geography of Dallas, TX shows that the optimal arrangements can substantially improve police efficiency as measured by the increased number of incidents within an acceptable service distance, and by the reduction in total response distance. Moreover, variations in the formulation can be used to generate solutions representing maximal backup coverage, and by solving a set of problems that maximize backup coverage with a constraint on maximal coverage, good solutions for both objectives can be found.

Although patrol boundaries have historically been fixed for extended periods of time, the ability to treat patrol areas as flexible spatial entities is increasingly demanded by police administrators. If a substantial number of police officers are unavailable for duty due to illness, for example, or in the event of a major natural or man-made disaster, the existing patrol area boundaries may not be relevant. As Community Policing increasingly becomes integrated into mainstream police patrol strategies, new district designs may be needed to reflect the boundaries of cohesive communities (Skogan 2004). The Police Patrol Area Covering (PPAC) Model allows police administrators to leverage their existing Geographic Information Systems technology, in combination with maximal coving location models to redesign patrol area boundaries based on conditions that they specify. PPAC allows patrol areas to be a flexible tool for providing police service.

With the increasing acceptance of GIS as an aid to police work, practitioners have begun to demand more robust quantitative tools. This research represents one contribution toward this goal, and it is hoped that the PPAC model and its implementation will be able to help police departments to better serve their population and use their resources more efficiently.

Acknowledgements Thanks to Brian J. L. Berry, Ron Briggs, Tom Cova, several reviewers at the North American Regional Science Conference, and anonymous reviewers for NETS for providing comments and suggestions on earlier versions of this paper.

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